B.Sc. IInd Year
Advanced Calculus
(Paper – III)
Note: 04

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Evolute of a curve in form of Envelope:

Introduction and Definition:– evolute is defined as the envelope of the normals of the given curve. Therefore the normal of a curve touches the evolute. And curve is known as envolute of evolute curve.

Locus of P (centre of curvature) is envelope of family of normals.

The evolute of a curve is envelope of its normal for centre of curvature is a point of intersection of P of two consecutive normals in limit as variation of parameter (taking parametric eqn of curve) tends to zero.

Example:– Find the evolute of the curve $x^2 - y^2 = a^2$

Solution:– We have $x^2 - y^2 = a^2$ ——— (I)

its parametric eqn is given by

$$x = a\sec\phi \,;\ y = a\tan\phi \quad \text{——— (II)}$$

i.e. $(a\sec\phi, a\tan\phi)$ always lies on curve (I).

from (I); we have

$$2x - 2y\frac{dy}{dx} = 0 \quad \left(\begin{array}{c}\text{Differentiating (I)}\\ \text{w.r.to } x\end{array}\right)$$

$$\Rightarrow 2y\frac{dy}{dx} = 2x \Rightarrow \frac{dx}{dy} = \frac{y}{x}$$

$$-\left(\frac{dx}{dy}\right)_{(a\sec\phi,\, a\tan\phi)} = -\frac{a\tan\phi}{a\sec\phi} = -\sin\phi \quad \text{——— (III)}$$

Eqn of normal at $(a\sec\phi, a\tan\phi)$ of eqn is given by $(y - a\tan\phi) = -\sin\phi(x - a\sec\phi)$

$$\Rightarrow x\sin\phi + y - 2a\tan\phi = 0 \quad \text{--- (iv)}$$

Here ϕ is a parameter.

Differentiating (iv) w.r.t. ϕ, partially; we have

$$x\cos\phi + 0 - 2a\sec^2\phi = 0$$

$$\Rightarrow x\cos^3\phi = 2a \Rightarrow \cos\phi = \frac{(2a)^{1/3}}{(x)^{1/3}}$$

$$\left.\begin{aligned} \Rightarrow \sin\phi &= \frac{\{(x)^{2/3} - (2a)^{2/3}\}^{1/2}}{(x)^{1/3}} \\ \text{and } \tan\phi &= \frac{\{(x)^{2/3} - (2a)^{2/3}\}^{1/2}}{(2a)^{1/3}} \end{aligned}\right\} \text{--- (v)}$$

Using eqn (v) in eqn (iv); we have

$$\frac{x\{(x)^{2/3} - (2a)^{2/3}\}^{1/2}}{(x)^{1/3}} + y - 2a\frac{\{(x)^{2/3} - (2a)^{2/3}\}^{1/2}}{(2a)^{1/3}} = 0$$

$$\Rightarrow \{(x)^{2/3} - (2a)^{2/3}\}^{\frac{3}{2}} = -y$$

squaring both sides & taking cube roots and arranging the terms; we have

$$(x)^{2/3} - (y)^{2/3} = (2a)^{2/3} \quad \text{--- (vi)}$$

which is required evolute.

Exercise :- Find the evolute of the curves

(i) $y^2 = 4ax$ (ii) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(iii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (iv) $x^2 + y^2 = a^2$